14p

NASA TECHNICAL TRANSLATION NASA TTF-14,774

NUMERICAL EXPERIMENTS ON INVERSION OF SCATTERING INDEX BY THE METHOD OF STATISTICAL REGULARIZATION

V. A. Gashko, L. S. Turovtseva, V. F. Turchin and K. S. Shifrin

(NASA-TT-F-14774) NUMERICAL EXPERIMENTS ON INVERSION OF SCATTERING INDEX BY THE METHOD OF STATISTICAL REGULARIZATION (Kanner (Leo) Associates) ~~18~~ 14 p HC $4.00 CSCL 12A G3/19

N74-20172

Unclas 32705

Translation of
"Chislennyye Eksperimenty po Obrashcheniyu Indikatris Rasseyaniya Metodom Statisticheskoy Regulyarizatsii,"
In: Optika okeana i atmosfery (Optics of the Ocean and Atmosphere), Edited by K.S. Shifrin, Leningrad, "Nauka" Press, 1972, pp 94-103

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 SEPTEMBER 1973

STANDARD TITLE PAGE

1. Report No. NASA TT F-14,774	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle NUMERICAL EXPERIMENTS ON INVERSION OF SCATTERING INDEX BY THE METHOD OF STATISTICAL REGULARIZATION		5. Report Date September 1973 6. Performing Organization Code
7. Author(s) V. A. Gashko, L. S. Turovtseva, V. F. Turchin, K. S. Shifrin		8. Performing Organization Report No. 10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates P. O. Box 5187 Redwood City, California 9063		11. Contract or Grant No. NASw-2481 13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address NASA, Washington, D.C. 02546		14. Sponsoring Agency Code

15. Supplementary Notes Translation of "Chislennyye Eksperimenty po Obrashcheniyu Indikatris Rasseyaniya Metodom Statisticheskoy Regulyarizatsii," In: Optika okeana i atmosfery (Optics of the Ocean and of the Atmosphere), Edited by K.S. Shifrin, Leningrad, "Nauka" Press, 1972, pp 94-103

16. Abstract The present work is a continuation of an earlier work on restoration of the spectrum of particle sizes by the method of statistical regularization. This work presents a more detailed study of the restoration of the particle distribution function with various errors in index, various areas of index sector angle used in inversion and various forms of the distribution of function, in order to formulate optimal requirements for experimental measurements.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unlimited. Unclassified

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
None	None	14	4.00

NUMERICAL EXPERIMENTS ON INVERSION OF SCATTERING INDEX BY THE METHOD OF STATISTICAL REGULARIZATION

V. A. Gashko, L. S. Turovtseva, V. F. Turchin and K. S. Shifrin

Statement of the Problem

/94

When it is possible to limit ourselves to one-time scattering, the problem of inversion of the index is reduced to the solution of an integral equation of the first kind

$$\overline{J}(\beta)=\int_0^\infty J(\beta,\ \rho)\,\varphi(\rho)\,d\rho, \tag{1}$$

where ρ is the diffraction parameter ($\rho = 2\pi a/\lambda$); $\phi(\rho)$ is the distribution function of particles by dimensions; $J(\beta, \rho)$ is the light scattering index on a single particle of radius $a = \rho\lambda/2\pi$; $\overline{J}(\beta)$ is a polydisperse index (determined experimentally); λ is the wavelength of the light.

The problem of determination of function $\phi(\rho)$ from $J(\beta, \rho)$ and $\overline{J}(\beta)$ is improper. The slight inaccuracies in determination of $\overline{J}(\beta)$ and $J(\beta, \rho)$, unavoidably arising due to errors in measurement or calculations, lead to great errors in $\phi(\rho)$. Earlier, the problem of inversion of the index was studied in detail [1-5] under certain assumptions concerning the kernel of equation $J(\beta, \rho)$. These kernels can be used only when scattering experiments /95 are specially arranged. However, the required type of experiment can not always be performed. It is therefore important to avoid these limitations and study the solution of equation (1) in its general statement.

Methods developed recently for solution of improper inverse problems allow us to approach the problem of inversion of the index from a common position.

Restoration of the particle dimension spectrum by the method of statistical regularization was first performed in [6]. The present work is a continuation of this work. Here we study the restoration of the particle dimension distribution function in more detail, with various errors of the index, various areas of index sector angles used in inversion and various forms of the distribution of function itself, in order to formulate optimal requirements for the experimental measurements.

The Method of Inversion

To solve the problem of determining the particle dimensions distribution function, we used the method of statistical regularization, developed in [7, 8].

The formulation of the method of statistical regularization naturally includes the stochastic nature of the process of scattering of light. Furthermore, it has one more valuable quality, in that it allows the desired function $\phi(\rho)$ to be restored, even if there are no statistical data on this function. It is this quality which is decisive in selecting the method of index inversion.

The problem is that the form of the function to be restored is unknown when an experimental index is inverted. In order to regularize the solution of the problem (eliminate physically senseless solutions), certain additional conditions must be imposed. The problem consists in proper selection of these conditions, since the selection of supplementary conditions actually determines the solution of the problem. The conditions placed on the desired function may express its membership in the statistical set of functions defined, for example, by the correlation matrix, mathematical expectation and other statistical characteristics of the set. However, one must be sure that the statistics used for regularization are adequate to the statistics to which the desired function actually belongs.

The use of inadequate ("foreign") statistics leads to an increase in the error of restoration, and the form of the

distribution restored may be greatly distorted. (They would tend, /96
for example, toward the mean introduced, i.e., would tend to correspond to the assigned statistics.) The production of original (particularly exotic) distributions is impossible with this approach.

However, we know that the restored function represents a specific physical quantity (probability density for particles of a given dimension) and is continuous. The continuity means that there is a more or less good correlation of the values of the desired function with similar ρ, depending on its smoothness (oscillation).

The method of statistical regularization [8] allows the desired function to be restored on the basis of information on its smoothness, the degree of smoothness being determined from the results of measurements of the index.

Calculations were performed on the VESM-4 computer in a closed cycle: distribution function $\phi(\rho)$ was assigned; the right portion was calculated in equation (1)[1]; the random error was added to the values of $J(\beta)$ produced; using the value $\bar{J}(\beta)$ produced, imitating the result of measurement of the index, $\phi(\rho)$ was restored according to an algorithm described in [8]. The results produced were compared with the initial function.

Initial Data and Calculation Conditions

The initial data for calculation were taken on the basis of certain considerations.

Selection of form of distribution function. The numerical experiments described below were performed with five different initial distribution functions ϕ_I-ϕ_V. Functions ϕ_I-ϕ_{IV} are single-mode distributions. In selecting them, we considered the fact that in most cases the spectrum of particles in an actual

[1]The values of $J(\beta, \rho)$ were taken from [9] for water drops and $\lambda = 0.712$ μ.

atmosphere can be described by means of the gamma distribution $\rho^{\mu}\exp(-\gamma\rho)$ [10]. Function ϕ_I differs somewhat from the gamma distribution with parameters $\mu = 2$, $\gamma = 2$ in that it rises in the area of slight ρ, i.e., the number of small particles in increased (Figure 1).

In most cases in a real atmosphere we encounter not single-peak, but rather more complex distributions with two and three peaks.

We therefore restored a three-mode arbitrary distribution function $\phi_V(\rho)$.

As was noted in [6], the distribution function in the area of small ρ is restored significantly worse than in the area of large ρ. This is related to the fact that the scattering index $\overline{J}(\beta)$ /98 carries information basically on the large particles. Information on the fine particles (dimensions less than a certain ρ_{min}) is very sparse and falls within the level of errors.

If we decrease the number of large particles, the relative contribution of light scattered on small particles to the scattering index increases, and the small-particle end of the distribution function is better restored. In order to check this assumption, calculations were performed with functions ϕ_{III} and ϕ_{IV}, produced from function ϕ_I by lowering the right end.

Selection of an error value. A 10% experimental accuracy is permissible in measurements of the scattering index in a free atmosphere under field conditions. For measurements in Cuvettes (under laboratory conditions), accuracy can be increased to 5, 3 or even 1%. However, in order to determine the general regularities of restoration, it is interesting to study restoration with even higher accuracies of assignment of the polydisperse index -- 0.1 and 0.05%. Model calculations were performed for these six levels of error in the value of $\overline{J}(\beta)$.

Selection of area of angles and number of points of index. In recording the scattering index in an actual atmosphere with

a standard nephelometer, angles of 10 to 160° were accessible. Therefore, model inversions must be performed within this range of angles.

/97

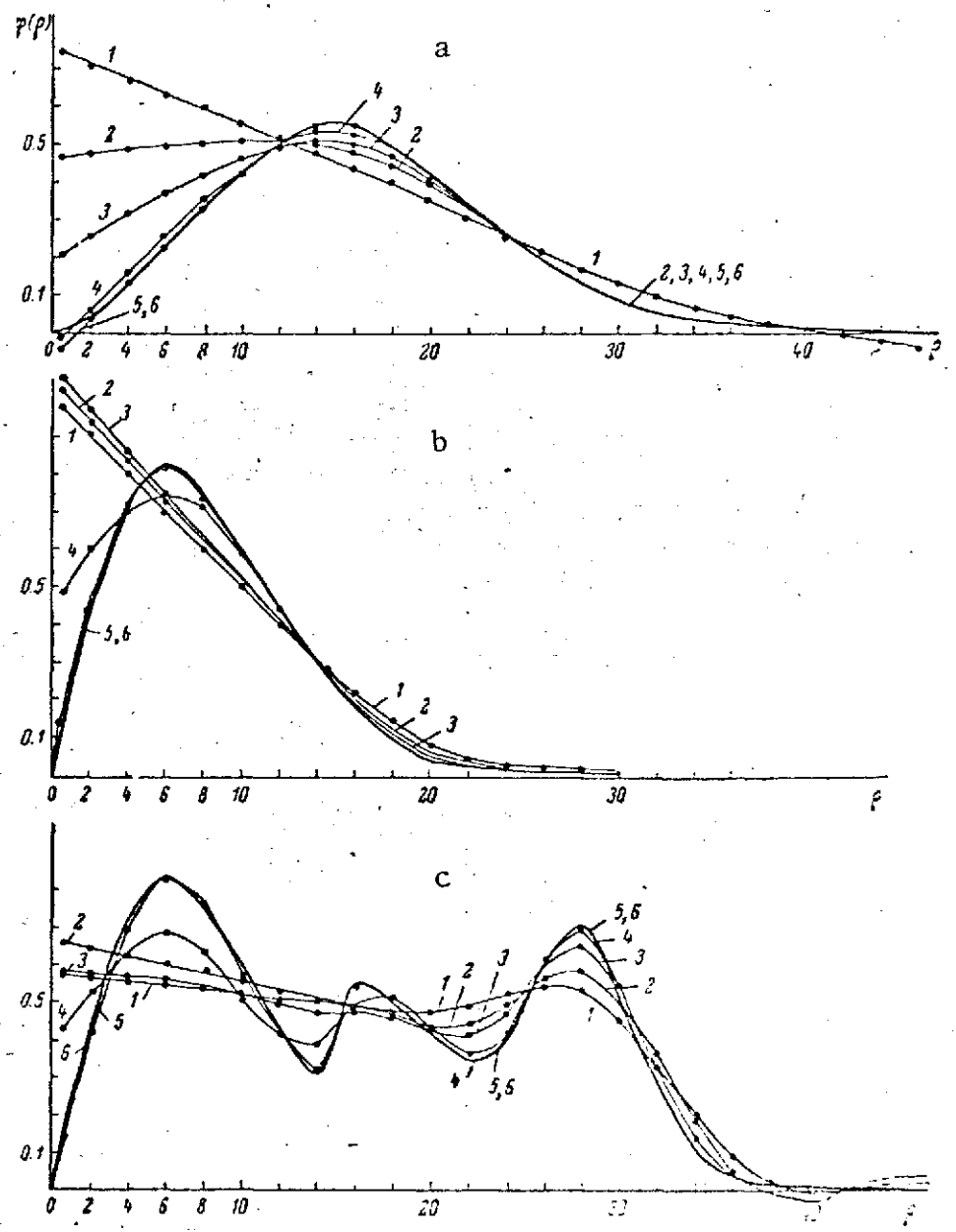

Figure 1. Restoration of Functions ϕ_I(a), ϕ_{II}(b), ϕ_V(c) with Mean Square Error: 1, 10; 2, 5; 3, 3; 4, 1; 5, 0.1; 6, 0.05%

However, as we know [4, 5], the greatest information on the medium and large particle area in the distribution curve is carried by measurements of the index made at slight angles. It is when the low-angle sector of the index is inverted that good restoration of large particles can be achieved. The small angle device, developed specially for these measurements, allows the index to be recorded in areas of 6 minutes, 0.1°. Calculations including this area of angles were performed in versions 2, 5, 7 and 8 (see Table).

Version Number	m	β°	γ_i				
			I	II	III	IV	V
1	31	10 (5) 160	+	+	+	+	+
2	31	0.1, 0.5, 1, 2, 5 (5) 130	+	—	—	—	—
3	20	10 (5) 105	+	—	—	—	+
4	20	65 (5) 160	+	—	—	—	—
5	12	0.1, 0.5, 1, 2, 5 (5) 35	+	+	—	—	+
6	12	10 (5) 65	+	+	—	—	+
7	12	0.1, 0.2, 0.5, 1, 2, 5, 135 (5) 160	+	—	—	—	+
8	6	0.1, 0.2, 0.5, 1, 2, 5	+	—	—	—	—

It is very important to determine the minimum number of points /99 on the index necessary to produce good restoration. To do this, we performed calculations with different numbers of points: 31, 20, 12 and even 6.

The calculations were performed for 19 cases, marked by the plus sign (+) in the table. All versions were calculated for six values of error. We thus studied the total of 19 x 6 = 114 cases.

Analysis of Results

Before going over to analysis of the results, let us recall that in the method of inversion we are using, the function restored is the mean function of the statistical set of regularized solutions. The error of restoration is defined as the mean square deviation of the function of this statistical set from the mean.

Comparing $\phi(\rho)$ produced under various restoration conditions (various sectors of index, various accuracies of determination of index, various numbers of measurements of $\overline{J}(\beta)$, we compare the mean

functions of the statistical sets of regularized solutions among themselves (and with the true function $\phi(\rho)$). We will discuss whether $\phi(\rho)$ is restored better or worse under various calculation conditions, comparing the restored functions $\phi(\rho)$ (mean function of statistical set) with the true $\phi(\rho)$, although their differences are frequently within the limits of experimental error. These errors are noted by the vertical sectors on certain figures.

Restoration with various errors. Figure 1 shows the results of calculations for the first version of functions ϕ_I, ϕ_{II} and ϕ_V. As was already noted [6], in restoring the index the most difficult portion of the problem is proper restoration of the fine-particle area of the distribution function. The contribution of fine particles can be considered only by an index measured with such high accuracy that the error of measurement is less than the illuminance of the small particles. An index measured with fixed accuracy can properly restore the distribution curve $\phi(\beta)$ only down to a certain ρ_{min}, i.e., to particles of a size such that their contribution to the scattering index is greater than the error with which the index is known.

Figure 1 shows the restored distribution functions with fixed ϕ_I and ϕ_{II}. As we see, proper restoration of the small-particle area of curve ϕ_{II} requires greater accuracy of determination of $\overline{J}(\beta)$ than when curve ϕ_I is restored. This is explained by the fact that distribution ϕ_{II} describes smaller particles than ϕ_I. /100
Proper restoration of the left end of ϕ_I can be achieved with an accuracy of $\overline{J}(\beta)$ such that the contribution of particles measuring ρ_{min} = 12-13 is considered, whereas proper restoration of ϕ_{II} requires much greater accuracy (ρ_{min} = 3-4). Therefore, measurements performed, for example, with an error of 1%, can achieve proper restoration of ϕ_I, whereas proper restoration of ϕ_{II} requires inversion of an index measured with higher accuracy (0.1, 0.05%).

We restored the three-mode distribution function ϕ_V. As we can see (Figure 1), the index "measured" with an accuracy of 10%

allows proper restoration of only the large-particle branch of the right maximum. Measurements made with accuracies of 3 and 5% properly define the position of the right maximum. The other two maxima are not revealed by the restored curves.

An index "measured" with an accuracy of 1% can, as we see, be used to restore all free maxima. The right maximum is restored quite near the true position. Only measurements performed with an accuracy of 0.1% and higher can restore all details of the two three-mode particle dimension distribution function. As the accuracy is increased, first the large-particle, then the small-particle areas of the distribution curve "appear."

Restoration of $\phi(\rho)$ from various sectors $\bar{J}(\beta)$. Let us study the restoration of $\phi(\rho)$ from index $\bar{J}(\beta)$, assigned in various areas of angles β. Figure 2a, b, c shows results of inversion of indices calculated for function ϕ_I and assigned with errors of 10, 3 and 1% respectively. Figure 3a, b, c shows the same for function ϕ_V. When the inversion of the various sectors of the index is compared with a fixed value of m (m = 31, 20, 12), as would be expected, better restoration of the large particles is observed with the small-angle index areas. Increasing $\bar{m}$ increases restoration quality.

The calculations performed for m = 6 (version 8) showed that this number of points of $\bar{J}(\beta)$ is insufficient for good restoration of the entire distribution function, even if they are taken in the low-angle area of the index (0.1-5°) and measured with high accuracy. Function $\phi(\rho)$, restored using 6 points on the index, does not describe the small-particle area, even with maximum area of determination of $\bar{J}(\beta)$ (at 0.05%), whereas the large-particle area of the distribution is described rather well with areas of 3 and 1% (Figure 2b, c) or less. Restoration using $\bar{J}(\beta)$ fixed /103
with an error of 10% where m = 6 transmits the course of the true $\phi(\rho)$ quite approximately.

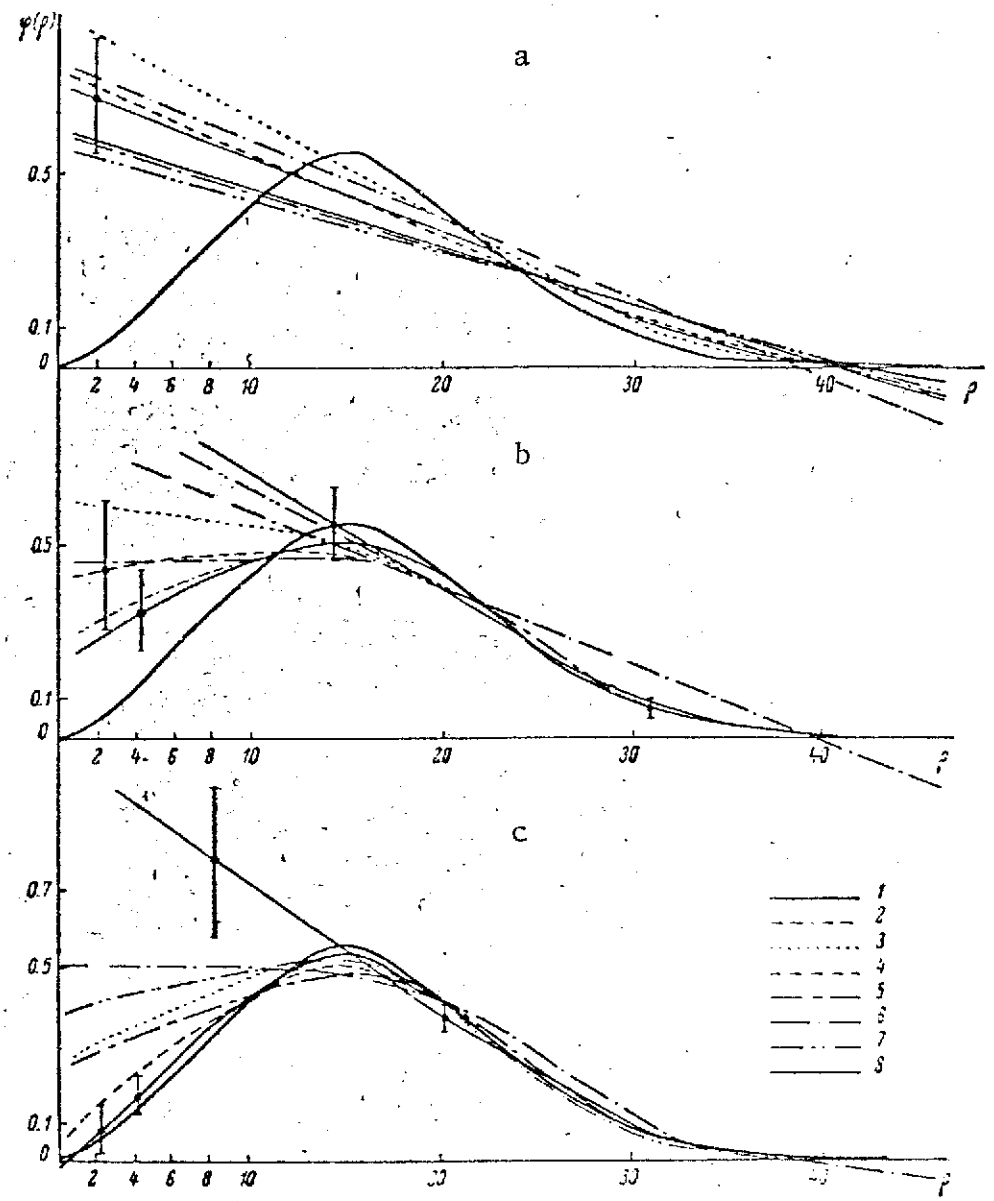

Figure 2. Restoration of Function ϕ_I with Mean Square Error: a, 10; b, 3; c, 1% Using Various Index Sectors. 1-8, Versions.

Restoration of function $\phi(\rho)$ of various forms. As we explained before, the small-particle area of the distribution curve is restored less well than the large-particle area. The information on large particles contained in the scattering index "blocks out" the information on the small particles. Obviously,

if we decrease the number of large particles, while leaving the small area of the distribution curve unchanged, restoration of the small-particle end should improve.

/102

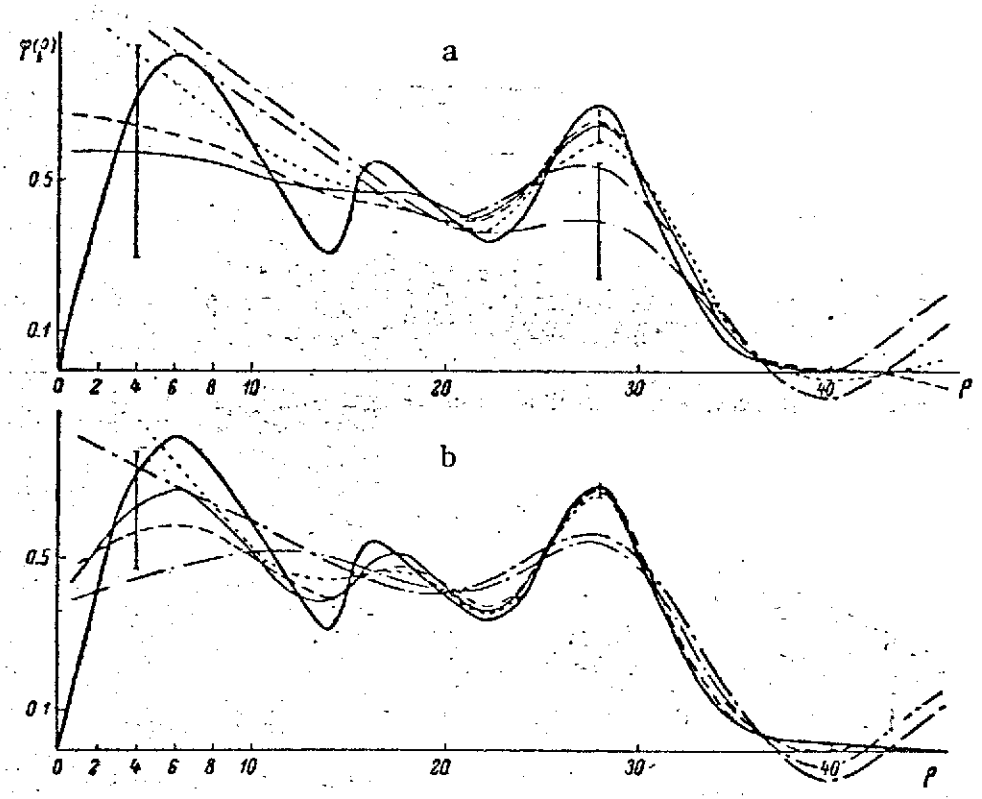

Figure 3. Restoration of Function ϕ_V with Mean Square Error a = 3%, b = 1% Using Various Index Sectors. 1, 3, 6, Same as on Figure 2.

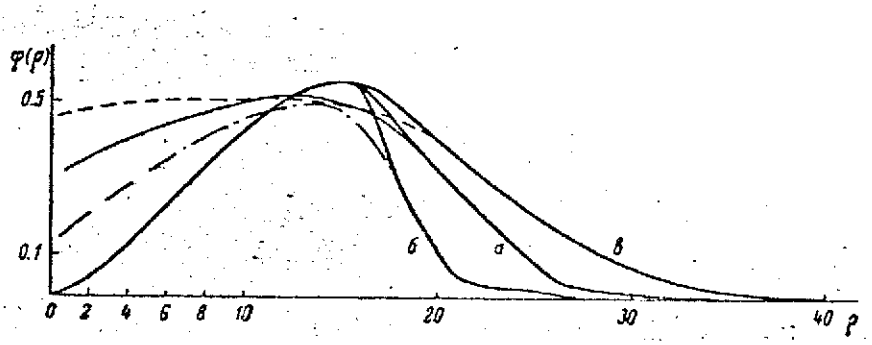

Figure 4. Restoration of Distribution Function ϕ_I (a), ϕ_{III} (b) and ϕ_{IV} (c) with Mean Square Error 5%

This was confirmed by our experiments, the results of which were produced by inversion of indices fixed with an accuracy of 5%, as shown on Figure 4. We see that the restoration of the small fraction is improved.

REFERENCES

1. Shifrin, K. S., "Calculation of a Class of Defined Integrals," Trudy Vses. Zaochnogo Lesotekhn. In-ta, Leningrad, No. 2.
2. Shifrin, K. S., "Optical Studies of Cloud Particles," Issledovaniye Oblakov, Osadkov i Grozovogo Elektrichestva [Studies of Clouds, Precipitation and Storm Electricity], Leningrad, 1957.
3. Shifrin, K. S., A. Ya. Perel'man, "Inversion of an Index for 'Soft' Particles," DAN SSSR, Moscow, 1964, Vol. 158, No. 3.
4. Shifrin, K. S., Vasseyaniye Sveta v Mutnoy Srede [Scattering of Light in a Turbid Medium], Leningrad, GITTL Press, 1951.
5. Shifrin, K. S., "The Significant Error of Scattering Angles," Izv. AN SSSR, Fiz. Atm. i Okeana, Moscow, 1966, No. 9.
6. Shifrin, K. S., V. F. Turchin, L. S. Turovtseva, V. A. Gashko, Izv. AN SSSR, Fiz. Atm. i Okeana, Moscow, 1972, No. 12.
7. Turchin, V. F., "Solution of the Fredholm Equation of the First Kind in a Statistical Set of Smooth Functions," Zh. Vychislit. Matem. i Matematich. Fiziki, 1967, Vol. 7, No. 6.
8. Turchin, V. F., V. Z. Nozik, "Statistical Regularization of the Solution of Improper Problems," Izv. AN SSSR, Fiz. Atm. i Okeana., Moscow, 1969, Vol. 5, No. 1.
9. Shifrin, K. S., I. L. Zel'manovich, Tablitsy po Svetorasseyaniyu [Table on Light Scattering], Leningrad, 1968, Vol. 2.
10. Shifrin, K. S., Ye. A. Polyakova, "Calculation of Microstructures," Trudy GGO, Leningrad, 1953, No. 42.